

Study of high resolution imaging algorithm based on compressed sensing imaging system

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Abstract. Affected by point spread function, the resolution of images captured by existing compressed sensing imaging system is always degraded. In order to improve the resolution of the image, the parameters of point spread function are substituted into the compressed sensing algorithm to obtain high-resolution images which break the resolution limit of the system. Then the proposed method is compared with the traditional method through numerical simulation. Results show that the proposed algorithm can effectively enhance the imaging resolution and obtain super-resolution image data. Finally, the recovery results are studied at the presence of measurement error of the point spread function, which indicate that super-resolution images can still be acquired when measurement error is considered. However, the signal-to-noise ratio of images deteriorates because of the error.

Key words. Compressed sensing, Super-resolution, Point spread function, Signal-to-noise ratio.

1. Introduction

Compressed sensing imaging system is a new type of imaging method, which randomly modulates the light intensity information obtained by imaging lens using modulation system, detects the total light intensity using single point probe, and finally acquires the 2-Dimensional (2D) information of the imaging scene from the reverse analysis of data which is obtained from compressed sensing algorithm. The Digital Micro-mirror Devices (DMD) are always deployed by the modulation system. Signals are still successfully recovered by the compressed sensing algorithm

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even if they are sampled at a frequency lower than what is defined by the sampling theorem. The development of compressed sensing algorithm has paved the way for the realization of compressed sensing imaging system. In 2007, the first compressed sensing imaging camera was invented by Rice University of the United States [1]. The theory of imaging was analysed and the corresponding experimental research was carried out. As a special imaging system, the development of compressed sensing imaging system was relatively slow in the beginning due to the constraint set by the sampling rate of the modulation and data acquisition system [2]. However, with the rapid advancement of high-speed data acquisition and processing, compressed sensing imaging system has again attracted the attention of scholars recently. In particular, the 3-Dimensional (3D) scene imaging using compressed sensing imaging system becomes available with the introduction of active laser illumination. Now there are generally two ways to implement 3D imaging using laser-illuminated compressed sensing imaging system [3,4]: Solution One is firstly obtaining the distance data from the echo signal of the scene, then inversely deducing the 3D information of the scene; Solution Two is firstly deploying several single-pixel detectors for imaging from different angles, then making use of data acquired by these detectors to inversely deduce the 3D information of the scene. In 2013, the first imaging method was adopted in the collaborative work of University of Rochester and Shanghai Institute of Optics and Fine Mechanics to obtain the 3D information of the imaging scene, and the corresponding research results were published [4]. In the same year, the second imaging method was adopted by researchers in University of Glasgow for the experimental study of 3D information, and the relevant research results was published on "Science" [3]. The 3D imaging technology utilizing compressed sensing imaging system has attracted the attention of an increasing number of domestic and foreign researchers [5]. Moreover, researchers in the University of Glasgow also studied the multi-spectral imaging technology based on the theory of compressed sensing imaging, and announced their experimental results [6].

Following the improvement on the quantity of information, people start to set new requirements on the quality of images [7]. For example, the resolution of the image as an important evaluation parameter of the imaging system, was not paid much attention to in compressed sensing imaging[8]. Modulation is required in compressed sensing for the scene information obtained from the optical acquisition system [9]. But the resolution of the scene captured by the optical acquisition system will degrade due to the limitation of many factors e.g. the relative motion between the object and the imaging system. Therefore, the paper started from the acquisition process of compressed sensing imaging, and considered the factors causing resolution degradation in the reverse analysis of 2D information, so as to obtain high-resolution image information.

2. Compressed sensing imaging system

The acquisition process of compressed sensing imaging system is shown in Fig. 1.

The original scene is captured by the imaging optical system and modulated by

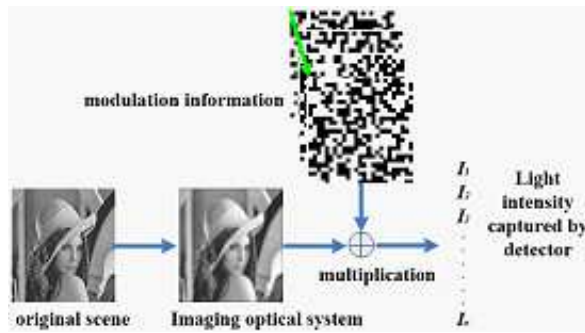


Fig. 1. Compressed sensing imaging system

the modulation system. The single-pixel detector obtains the total modulated light intensity.

Fig. 2 elaborates the optical principle of compressed sensing imaging system. The n -th data retrieval process of compressed sensing imaging system can be represented by the following matrix:

$$I_n = \sum_{x,y} b_n(x,y) o'(x,y) = \sum_{x,y} b_n(x,y) [h(x,y) \otimes o(x,y)] \quad (1)$$

Among them, $o(x,y)$ is the information of the original scene; $h(x,y)$ is the point spread function of the imaging system; $b_n(x,y)$ is the n -th acquired modulation information; I_n is the n -th light intensity captured by the single-pixel detector; (x,y) are the coordinates in the 2D space; and $o'(x,y) = h(x,y) \otimes o(x,y)$ is the scene information processed by the imaging optical system.

In the study of compressed sensing imaging system, the degradation in the resolution of the image is always overlooked. But actually the image tends to become blurred due to the influence of point spread function. This function is directly applied to the inverse analysis of the optical imaging system. The optical imaging system discussed here refers to the optical subsystem which projects the object onto DMD. Whereas the optical subsystem which collects the signal reflected from DMD does not affect the imaging resolution. However, in the study of high-resolution imaging, the point spread function of the optical subsystem must be considered. In this paper, the influence of the point spread function of the projecting optical subsystem is investigated in the inverse acquisition of the image, in order to obtain the super-resolution image information.

Defining the number of pixels of the image as m , the repeated acquisition process can be expressed in the following matrix form:

$$I = BHO \quad (2)$$

Wherein, B denotes the n -th modulation matrix with the dimension of $n \times m$; the s -th ($s < n$) row is the vector expression of the modulation information in the s -th data acquisition; H denotes the point spread function in matrix form; O denotes

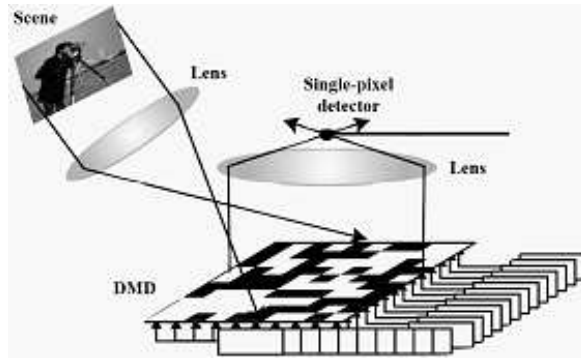


Fig. 2. Optical principle of compressed sensing system

the information of the original scene in matrix form; I is the column vector with n elements, each element indicates the intensity captured by the single-pixel detector in every data acquisition. It is assumed that the point spread function of the projecting optical subsystem is already given which makes $T = BH$. Substituting it into Equation(2) can get Equation(3):

$$I = TO \tag{3}$$

With I and T already known, the information of the original scene O can be obtained by solving the equation group above. The original scene contains m pixels i.e. m unknowns. In order to solve the m unknown numbers accurately, m times of measurement are needed and the modulation matrix for the m times of measurement should be independent. Thus the real-time performance of the imaging system is deteriorated. Given that in the space defined by certain base vectors, most of the scene in nature are sparse. The compressed sensing algorithm can still get accurate information of the scene with the number of samples n far less than m . So the compressed sensing algorithm is recommended for sparse or compressible signals, which have few non-zero elements in the space defined by certain base vectors [10]. Most of the signals in real life being transformed by a group of base vectors are sparse signals, such as signals after wavelet transform or discrete cosine conversion. The expression of this kind of signals is written below:

$$O = \psi\alpha \tag{4}$$

Wherein $\hat{\alpha} = \arg \min \|\alpha\|$ subject to $I = T\psi\alpha$ is the sparse transformation matrix. Substituting Equation (4) into Equation (3), the accurate information of $\hat{\alpha} = \arg \min \|\alpha\|$ subject to $I = T\psi\alpha$ can be obtained by solving $\hat{\alpha} = \arg \min \|\alpha\|$ subject to $I = T\psi\alpha$, i.e.:

$$\hat{\alpha} = \arg \min \|\alpha\| \text{ subject to } I = T\psi\alpha \tag{5}$$

Currently methods like matching tracking, interior point and gradient methods are generally deployed to find the solutions to Equation (5). Algorithms are applied

to obtain result $\hat{o} = \arg \min \|TV(O)\|$ *subject to* $I = TO$. Further solving Equation (4) can get the final image O of the object.

Generally in compressed sensing algorithm, sparse transformation is performed on signals. The sparse transformation matrix is used to solve the algorithm. In most cases, the gradient representation of discrete signals is sparse, which has been verified by a large number of experimental results. When the algorithm is solved using the gradient of discrete signal O , the solution model can be expressed as [12]:

$$\hat{o} = \arg \min \|TV(O)\| \quad \text{subject to } I = TO \quad (6)$$

Wherein TV represents the gradient value of the discrete signal.

3. Computer simulation

Optical systems always have defects, of which the impact on imaging systems can be described by point spread function (*psf*). The mathematical expression of the function is copied below:

$$psf = |F^{-1}(pe^{i\phi})|^2 \quad (7)$$

Wherein, F^{-1} denotes the Inverse Fourier Transform; p denotes the Pupil Function; and ϕ is the wave front aberration. Normally the point spread function affecting the imaging result of imaging systems can be expressed by Gaussian distribution, i.e.:

$$h_{psf} = \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) \quad (8)$$

Wherein σ is the standard deviation which determines the level of system degradation. The influence of point spread function on the imaging system are assessed by simulation according to the level of degradation incurred. The recovery results at the presence of measurement error of σ are also studied. Here only the point spread function of the projecting optical subsystem that is influential to the imaging quality is discussed.

The first group of images (Fig. 3) are the recovery results when the standard deviation is equal to 0.8. A1 is the original image with three strip sheets of different resolutions. It can be seen from the recovery result that the resolution of the images taken by traditional method degrade under the influence of point spread function, while images of super-resolution is obtained when the parameters of point spread function is substituted into the algorithm solving. In addition, the images recovered by traditional method have low signal-to-noise ratio, while both the signal-to-noise ratio and resolution of the images obtained by the method introduced in this paper are significantly improved.

The second group of images (Fig. 4) are the recovery results when the standard deviation is equal to 1.2. The same conclusions can be obtained from the results. For both methods, the signal-to-noise ratio of images increases with the number of samples. But with the same number of samples, the proposed method achieves a

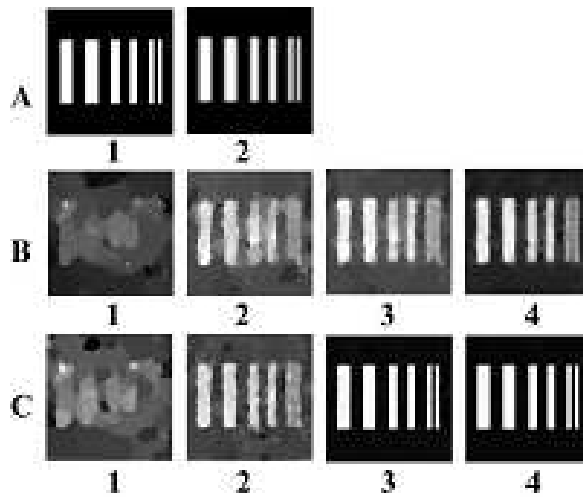


Fig. 3. Recovery results when standard deviation is equal to 0.8: A1 is the original image; A2 is the degraded image; B and C are the recovery results using traditional and proposed methods, respectively; 1, 2, 3 and 4 are the recovery results with 100, 300, 500 and 700 samples, respectively.

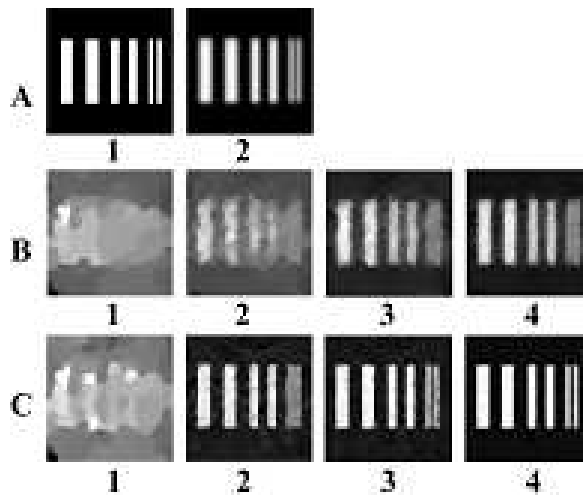


Fig. 4. Recovery results when standard deviation is equal to 1.2: A1 is the original image; A2 is the degraded image; B and C are the recovery results using traditional and proposed methods, respectively; 1, 2, 3 and 4 are the recovery results with 100, 300, 500 and 700 samples, respectively.

higher signal-to-noise ratio. Comparison of the above two groups of results reveals that with the same number of samples, the signal-to-noise ratio of the images recovered by the developed method deteriorates as the standard deviation of point spread function increases. To achieve a high signal-to-noise ratio, the number of samples needs to be increased corresponding to a point spread function with bigger standard

deviation. Still, the results of the two groups of experiment proved that the proposed method can effectively overcome the influence of the system point spread function, and finally obtain images of super resolution.

Practically, error is inevitable in the measurement of point spread function of the projecting optical subsystem. So the third group of experiment analyses the recovery results at the existence of measurement error of the point spread function. The standard deviation of the function in real case is assumed to be 0.8. Eight point spread functions with a standard deviation from 0.4 to 1.1 are deployed to recover the information of the image. Recovery results are shown in Fig. 5. It can be seen from the results that considering the error of point spread function, the method introduced in this paper still achieves much higher resolution of images than traditional method, and gets image information of super-resolution. However, the signal-to-noise ratio of images becomes worse at the presence of error, which tends to be even lower corresponding to bigger estimated error.

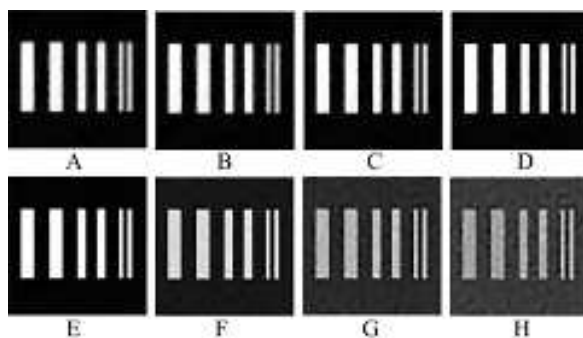


Fig. 5. Recovery results at the presence of error: A, B, C, D, E, F, G and H are the recovered results when the standard deviation of point spread function is equal to 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0 and 1.1, respectively. The number of samples is 800

4. Conclusion

Affected by point spread function, the resolution of images taken by compressed sensing imaging system reduces. So a new method is proposed to eliminate the influence of point spread function on image resolution by substituting it into the algorithm of compressed sensing. Numerical simulations are conducted to study the point spread functions of three different sizes. Results show that the new algorithm can effectively improve the image resolution and obtain super-resolution images. Finally the recovery results at the presence of error of point spread function are investigated. Using the proposed algorithm, images of super-resolution can still be acquired, but with a reduced signal-to-noise ratio because of the measurement error. In a word, the method developed in this paper effectively overcomes the influence of the system point spread function and obtains the image information of super-resolution.

References

- [1] T. CHEN, Z. W. LI, J. L. WANG: *Imaging system of single pixel camera based on compressed sensing*. Optics and precision engineering 15 (2012), No. 11, 2523–2530.
- [2] W. L. CHAN, K. CHARAN, D. TAKHAR: *A single-pixel terahertz imaging system based on compressed sensing*. Appl phys lett 93 (2008), No. 12, 230–237.
- [3] B. SUN, M. P. EDGAR, R. BOWMAN: *3D computational imaging with single-pixel detectors*. Science 340 (2013), No. 6134, 844–847.
- [4] G. A. HOWLAND, D. J. LUM, M. R. WARE, J. C. HOWELL: *Photon counting compressive depth mapping*. Opt express 7 (2013), No. 21, 23822–23837.
- [5] S. ZHANG, J. WANG, J. C. WANG: *Simple calculation method for three-dimensional imaging based on compressed sensing*. Acta optica sinica (2013), No. 1, 84–90.
- [6] D. SHI, J. HUANG, F. WANG: *Enhancing resolution of single-pixel imaging system*. Optical review 22 (2015) 308–314.
- [7] D. SHI, C. FAN, H. SHEN: *Reconstruction of spatially misaligned and turbulence degraded images*. Optics and lasers in engineering (2012), No. 50, 38–46.
- [8] X. L. LI, B. LI: *Review on progress of real-time THz sensing and imaging technology*. Laser & optoelectronics progress (2012), No. 9, 55–60.
- [9] E. J. CANDÈS, J. ROMBERG: *Robust uncertainty principles: exact signal reconstruction from highly incomplete frequency information*. IEEE transactions information theory 52 (2006), No. 2, 489–509.
- [10] W. Z. SHAO, Z. H. WEI: *Advances and perspectives on compressed sensing theory*. Journal of image and graphics 20, (2012), No. 1, 1–12.

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